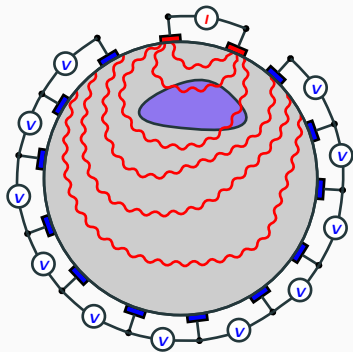


On the non-convexity issue in electrical impedance tomography

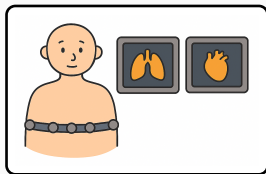
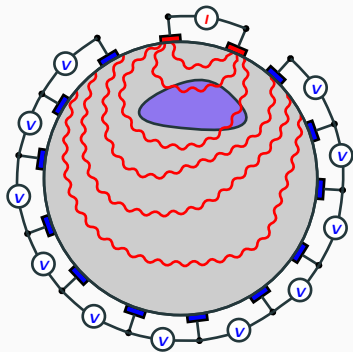
Romain Petit

joint work with Giovanni S. Alberti (University of Genoa)
and Clarice Poon (University of Warwick)

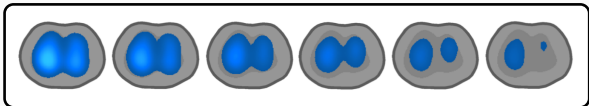
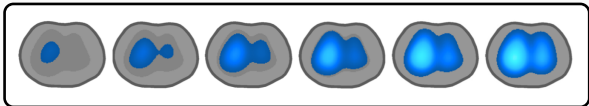
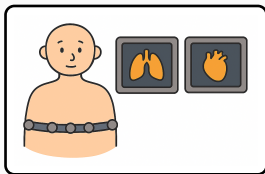
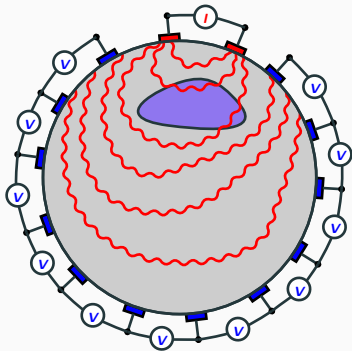
Electrical impedance tomography (EIT)



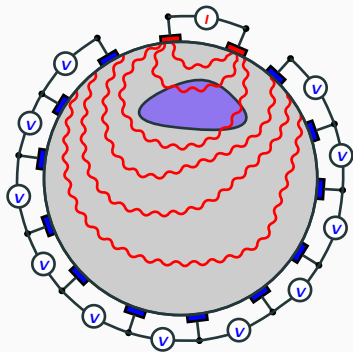
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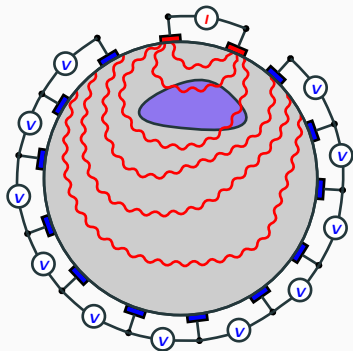
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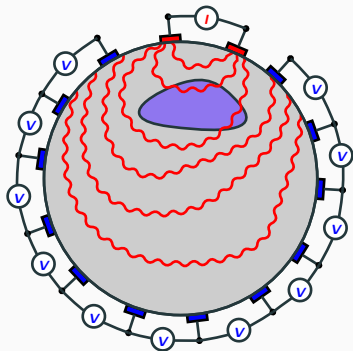
Electrical impedance tomography (EIT)



Model (Calderón problem)

- Introduced in [Calderón, 1980]

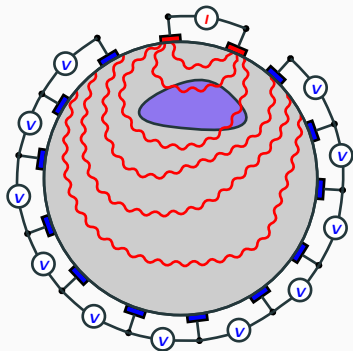
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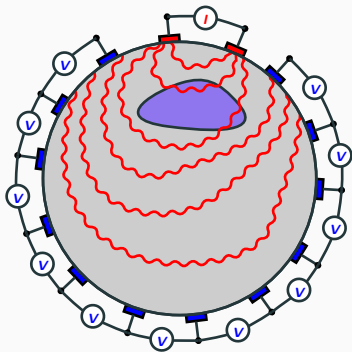
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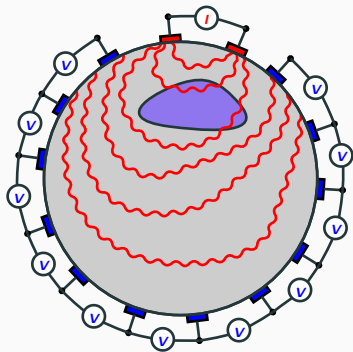
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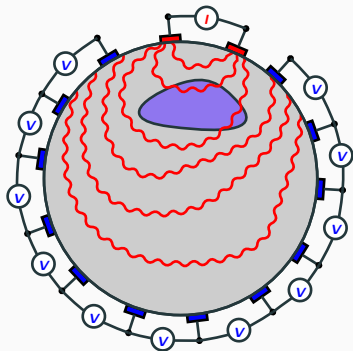
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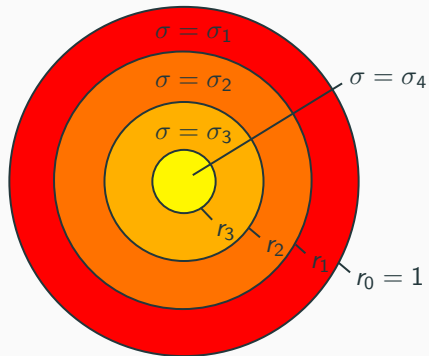
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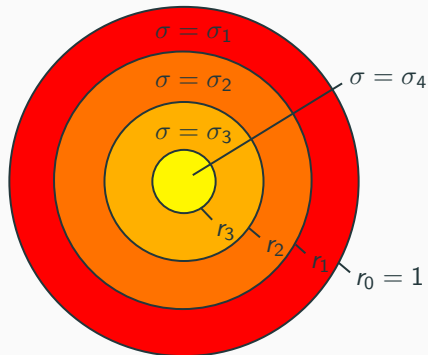
This talk

Investigate landscape
of $\sigma \mapsto \|\Lambda(\sigma) - y\|_2^2$
in **radial setting**

Radial setting



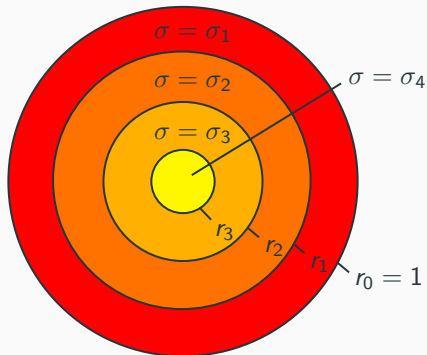
Radial setting



Forward map [Siltanen et al., 2000]

- If $g_j(\theta) = \cos(j\theta)$ then $\sigma \partial_\nu u|_{\partial\Omega} = \lambda_j(\sigma) g_j$.

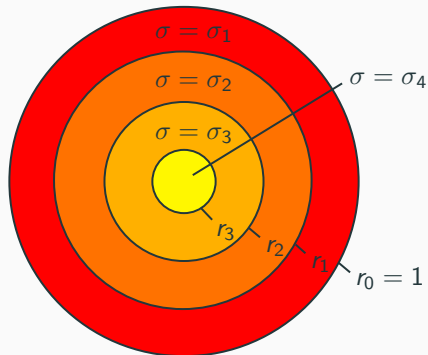
Radial setting



Forward map [Siltanen et al., 2000]

- If $g_j(\theta) = \cos(j\theta)$ then $\sigma \partial_\nu u|_{\partial\Omega} = \lambda_j(\sigma) g_j$.
- Define $\Lambda : \sigma \in \mathbb{R}^n \mapsto [\lambda_j(\sigma)]_{1 \leq j \leq m} \in \mathbb{R}^m$.

Radial setting



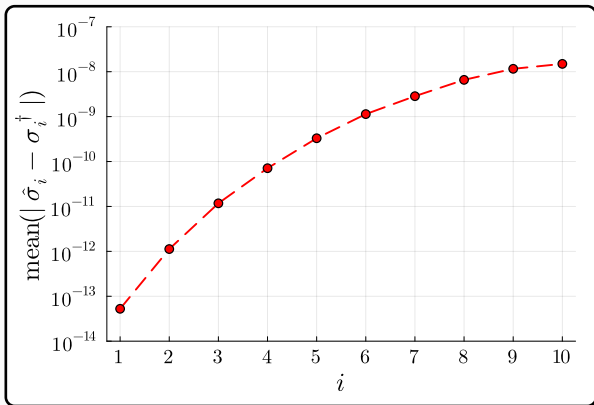
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- Define $\Lambda : \sigma \in \mathbb{R}^n \mapsto [\lambda_j(\sigma)]_{1 \leq j \leq m} \in \mathbb{R}^m$.
- Evaluate Λ by solving linear system (+ derivatives via autodiff)

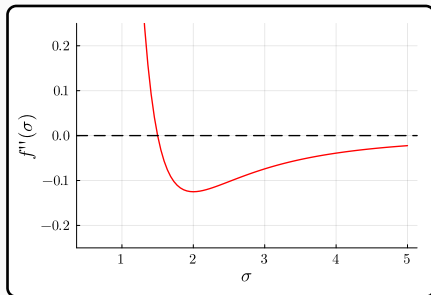
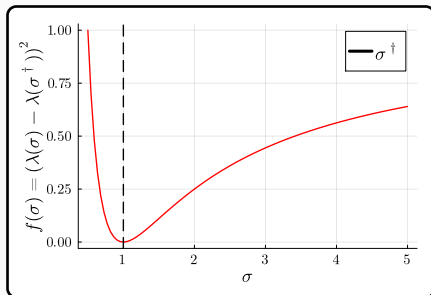
Ill-posedness

Setting

- Draw $\hat{\sigma}, \sigma^\dagger$ such that $\|\Lambda(\hat{\sigma}) - \Lambda(\sigma^\dagger)\|_\infty \leq 10^{-15}$.
- Compute mean error $|\hat{\sigma}_i - \sigma_i^\dagger|$ on each annulus



One-dimensional case



Identifiability and absence of bad critical points

Forward map $\Lambda : \sigma \mapsto (\lambda_j(\sigma))_{1 \leq j \leq m}$

Identifiability and absence of bad critical points

Forward map $\Lambda : \sigma \mapsto (\lambda_j(\sigma))_{1 \leq j \leq m}$

Conjecture

If $m \geq n$, the following holds.

- If $\Lambda(\sigma) = \Lambda(\sigma^\dagger)$ then $\sigma = \sigma^\dagger$.
- The unique critical point of $\sigma \mapsto \|\Lambda(\sigma) - \Lambda(\sigma^\dagger)\|_2^2$ is $\sigma = \sigma^\dagger$.

Identifiability and absence of bad critical points

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Partial results

- Full proof for $n = 2$ (proof of case $m > 2$ by Irène Waldspurger).
- Partial proof for $n > 2$ under a numerically verifiable criterion.

Content

- Implementation of forward map (+ derivatives via autodiff)
- Least squares approach
- Extensive benchmark against [Harrach, 2023] (convex programming)

The screenshot shows the GitHub repository page for `RadialCalderon.jl`. The page includes a sidebar with navigation links: Home, Getting started, Citation, Tutorials, Forward map, Convex nonlinear SGP, Least squares, Experiments, B postscript, Least squares vs nonlinear SGP, API reference, and References. The main content area displays the repository name, a description of the package for studying the Calderon problem, and a 'Getting started' section with a code block for installation. A 'Citation' section provides a preprint link. The footer mentions 'Powered by Documenter.jl and the Julia Programming Language'.

The screenshot shows the 'Tutorials / Least squares' page. It features a section titled 'Reconstruction via nonlinear least squares' with an introduction to the least squares approach. A mathematical formula for the functional $f: \sigma \mapsto \frac{1}{2} \|\Lambda(\sigma) - \Lambda(\sigma^1)\|_2^2$ is provided. The text explains the challenges of local convergence and the robustness of the iterative algorithm. A 'Setting' section shows a code block for initializing the problem. The 'Using NonlinearSolve.jl' section describes how to solve the equation $\Lambda(\sigma) = \Lambda(\sigma^1)$ using the `NonlinearSolve.jl` package.

Conclusion

Summary

- Mild non-convexity.
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Perspectives

- Tailored algorithms with better performance?
- Study landscape in noisy regularized setting.

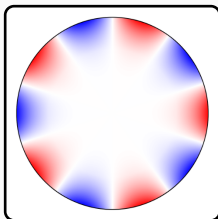
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Preprint

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radial Calderón problem

[arXiv:2507.03379](https://arxiv.org/abs/2507.03379)



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